# AXISYMMETRIC PROBLEM IN THE THEORY OF ELASTICITY FOR A HALF-SPACE WEAKENED BY A PLANE CIRCULAR CRACK

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The elastic equilibrium of a half-space, containing a circular crack, situated in a plane parallel to the boundary (Fig.1) is investigated. In the case of an axisymmetric load the problem is reduced to a system of dual integral equations, and then to a system of regular Fredholm equations. Some numerical results are obtained which are related to the stress concentration at the axial tension.

**1. Formulation of problem and its reduction to a system of dual integral equations.** We represent elastic displacements in the axisymmetric case by two harmonic functions of Papkovich-Neuber, $\Phi$ and $F$, in the following form (cf. for example [1]):

$$2Gu_r = -\frac{\partial\varphi}{\partial r} - z\frac{\partial F}{\partial r}, \quad 2Gu_z = (3-4\nu)F - \Phi - z\frac{\partial F}{\partial z}, \quad \Phi = \frac{\partial\varphi}{\partial z} \qquad (1.1)$$

In order to formulate the boundary conditions for the posed problem, we express by means of the functions introduced, the normal $\sigma_z$ and tangential $\tau_{rz}$ stresses

$$\sigma_z = 2(1-\nu)\frac{\partial F}{\partial z} - \frac{\partial\Phi}{\partial z} - z\frac{\partial^2 F}{\partial z^2}, \quad \tau_{rz} = \frac{\partial}{\partial r}\left[(1-2\nu)F - \Phi - z\frac{\partial F}{\partial z}\right] \qquad (1.2)$$

Let us divide the body in two domains (Fig.1): (1) layer $-h < z < 0$ and (2) half-space $0 < z < \infty$ and give functions $F$ and $\Phi$ in these domains the index 1 and 2. If we assume that the stresses on the surface of the crack and on the boundary of the body are specified, the boundary conditions may be written in the form

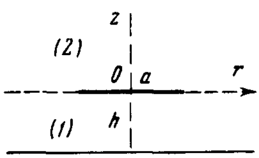

Fig. 1

$$\sigma_z|_{z=-h} = \sigma_0(r), \qquad \tau_{rz}|_{z=-h} = \tau_0(r)$$
$$\sigma_z|_{z=-0} = \sigma_1(r), \qquad \sigma_z|_{z=+0} = \sigma_2(r) \qquad (r < a) \qquad (1.3)$$
$$\tau_{rz}|_{z=-0} = \tau_1(r), \qquad \tau_{rz}|_{z=+0}| = \tau_2(r)$$

In addition, to pass through plane $z = 0$ at $r > a$, the values of the displacements and stress should be continuous. Then the indicated conditions are expressed by means of the functions $F_1$, $\Phi_1$, $F_2$, $\Phi_2$ in the following way:

$$\left[2(1-\nu)\frac{\partial F_1}{\partial z}-\frac{\partial \Phi_1}{\partial z}+h\frac{\partial^2 F_1}{\partial z^2}\right]_{z=-h}=\sigma_0(r) \qquad (1.4)$$

$$\left[(1-2\nu)F_1-\Phi_1+h\frac{\partial F_1}{\partial z}\right]_{z=-h}=S_0(r)=\int_\infty^r \tau_0(r)\,dr \qquad (1.5)$$

$$\left[2(1-\nu)\frac{\partial F_1}{\partial z}-\frac{\partial \Phi_1}{\partial z}\right]_{z=0}-\left[2(1-\nu)\frac{\partial F_2}{\partial z}-\frac{\partial \Phi_2}{\partial z}\right]_{z=0}=\begin{cases}0 & (r>a)\\ \sigma(r)-\sigma_1(r)-\sigma_2(r) & (r<a)\end{cases} \qquad (1.6)$$

$$[(1-2\nu)F_1-\Phi_1]_{z=0}-[(1-2\nu)F_2-\Phi_2]_{z=0}=\begin{cases}0 & (r>a)\\ S(r) & (r<a)\end{cases} \qquad (1.7)$$

$$\left[2(1-\nu)\frac{\partial F_1}{\partial z}-\frac{\partial \Phi_1}{\partial z}\right]_{z=-0}=\sigma_1(r),\qquad [(1-2\nu)F_1-\Phi_1]_{z=-0}=S_1(r)+ch \quad (r<a)$$

$$S(r)=\int_a^r[\tau_1(r)-\tau_2(r)]dr,\qquad S_1(r)=\int_0^r \tau_1(r)\,dr \qquad (1.8)$$

$$F_1\big|_{z=0}=F_2\big|_{z=0},\qquad \frac{\partial F_1}{\partial z}\Big|_{z=0}=\frac{\partial F_2}{\partial z}\Big|_{z=0}\qquad (r>a) \qquad (1.9)$$

$$\int_0^a \frac{\partial \Phi_1}{\partial z}\Big|_{z=0} r\,dr=\int_0^a \frac{\partial \Phi_2}{\partial z}\Big|_{z=0} r\,dr \qquad (1.10)$$

The relations (1.9), (1.10) guarantee the continuity of the displacements on the plane $z = 0$ at $r > a$ (cf. [2], page 41), and (1.10) allows, in addition, the determination of the constant $c$.

Therefore, the formulated problem is reduced to the determination of four functions $F_1$, $\Phi_1$, $F_2$, $\Phi_2$, that are harmonic in the region $-h < z < 0$ and $0 < z < \infty$, respectively, and satisfying conditions (1.4), (1.10), and also the requirements at infinity, which ensure proper behavior of displacements and stresses.

The unknown functions are sought in the form of the following integral equations:

$$F_1=\int_0^\infty[A\cosh\lambda(h+z)+B\sinh\lambda(h+z)]J_0(\lambda r)\frac{d\lambda}{\sinh\lambda h},\qquad F_2=\int_0^\infty Ee^{-\lambda z}J_0(\lambda r)\,d\lambda$$

$$\Phi_1=\int_0^\infty[C\cosh\lambda(h+z)+D\sinh\lambda(h+z)]J_0(\lambda r)\frac{d\lambda}{\sinh\lambda h},\qquad \Phi_2=\int_0^\infty Fe^{-\lambda z}J_0(\lambda r)\,d\lambda \qquad (1.11)$$

Conditions (1.4) and (1.5) allow one to obtain the two relations

$$D=2(1-\nu)B+\mu A-\sigma_0^\circ,\quad C=(1-2\nu)A+\mu B-\mu S_0^\circ \qquad (1.12)$$

Here

$$\sigma_0=\int_0^\infty \sigma_0(r)J_0(\lambda r)\,rdr,\qquad S_0=\frac{1}{h}\int_0^\infty S_0(r)J_0(\lambda r)\,rdr,\qquad \mu=\lambda h \qquad (1.13)$$

Analogously, from (1.6) and (1.7) we find

$$2(1-\nu)(A+B\coth\mu)-(C+D\coth\mu)-F+2(1-\nu)E=\sigma^\circ$$

$$(1-2\nu)(A\coth\mu+B)-(C\coth\mu+D)+F-(1-2\nu)E=\mu S^\circ$$

$$\sigma^\circ=\int_0^\infty \sigma(r)J_0(\lambda r)\,rdr,\qquad S^\circ=\frac{1}{h}\int_0^a S(r)J_0(\lambda r)\,rdr \qquad (1.14)$$

Relations (1.12) and (1.14) allow one to express all desired values by means of two functions of a parameter $\lambda$, in which case it is convenient to use the following:

$$M = A + B \coth\mu + E, \qquad N = A\coth\mu + B - E \tag{1.15}$$

Then, after some transformations, the remaining conditions (1.8) and (1.9) are reduced to the following system of dual integral equations:

$$\int_0^\infty MJ_0(\lambda r)\lambda d\lambda = 0, \qquad \int_0^\infty NJ_0(\lambda r)\, d\lambda = 0 \qquad (r > a) \tag{1.16}$$

$$\int_0^\infty \left[\frac{N}{2} - \left(\frac{1}{2} + \mu + \mu^2\right)e^{-2\mu} N - \mu^2 e^{-2\mu} M\right] J_0(\lambda r)\lambda d\lambda = f_1(r) \qquad (r < a) \tag{1.17}$$

$$\int_0^\infty \left[-\frac{M}{2} + \left(\frac{1}{2} - \mu + \mu^2\right) e^{-2\mu} M + \mu^2 e^{-2\mu} N\right] J_0(\lambda r)\, d\lambda = f_2(r) \qquad (r < a)$$

Here (1.18)

$$f_1 = \sigma_1 - \int_0^\infty \left\{(\bar\sigma + \mu\bar S)\frac{1 + \mu - \mu\coth\mu}{1 + \coth\mu} + (\coth\mu - 1)\,[(1 + \mu)\,\bar\sigma_0 + \mu^2 \bar S_0]\right\}\lambda J_0(\lambda r)\, d\lambda$$

$$f_2 = S_1 + \cosh - \int_0^\infty \left\{(\bar\sigma + \mu\bar S)\frac{1 - \mu + \mu\coth\mu}{1 + \coth\mu} + \mu(\coth\mu - 1)\,[(1 - \mu)\,\bar S_0 + \bar\sigma_0]\right\} J_0(\lambda r)\, d\lambda$$

We note also that relation (1.10) may not be represented in the form

$$\int_0^\infty M(\lambda) J_1(\lambda a)\, d\lambda = 0 \tag{1.19}$$

**2. Reduction of dual equations to Fredholm integral equations.** If we introduce new unknown functions $\varphi$ and $\psi$ by the relations

$$M(\lambda) = h\int_0^a \psi(t)\cos\lambda t dt, \qquad N(\lambda) = \frac{1}{\lambda}\int_0^a \varphi(t)(\cos\lambda t - \cos\lambda a)\, dt \tag{2.1}$$

then with the aid of Formula [3]

$$\int_0^\infty J_0(\lambda r)\sin\lambda t d\lambda = 0 \qquad (0 \leqslant t < r) \tag{2.2}$$

we can establish that Equations (1.16) are satisfied identically.

Further, if we substitute (2.1) in (1.17), we find the following equations at $r < a$:

$$\frac{1}{2}\int_0^a \varphi(t)\, dt \int_0^\infty J_0(\lambda r)(\cos\lambda t - \cos\lambda a)\, d\lambda - \int_0^a \varphi(t)\, dt \int_0^\infty h_1(\lambda)(\cos\lambda t -$$

$$- \cos\lambda a) J_0(\lambda r)\, d\lambda - \int_0^a \psi(t)\, dt \int_0^\infty g_1(\lambda)\cos\lambda t J_0(\lambda r)\, d\lambda = f_1(r) \tag{2.3}$$

$$-\frac{1}{2}\int_0^a \psi(t)\, dt \int_0^\infty J_0(\lambda r)\cos\lambda t d\lambda + \int_0^a \psi(t)\, dt \int_0^\infty g_2(\lambda)\cos\lambda t J_0(\lambda r)\, d\lambda +$$

$$+ \int_0^a \varphi(t)\, dt \int_0^\infty h_2(\lambda)(\cos\lambda t - \cos\lambda a) J_0(\lambda r)\, d\lambda = f_2(r) \tag{2.4}$$

Here

$$h_1 = (1/2 + \mu + \mu^2)\, e^{-2\mu}, \quad g_1 = \mu^3 e^{-2\mu}, \quad h_2 = \mu e^{-2\mu}, \quad g_2 = (1/2 - \mu + \mu^2) e^{-2\mu} \tag{2.5}$$

With the aid of Formula [3]

$$\int_0^\infty J_0(\lambda r) \cos \lambda t d\lambda = \begin{cases} 0 & (r < t) \\ (r^2 - t^2)^{-1/2} & (r > t) \end{cases} \tag{2.6}$$

$$J_0(\lambda r) = \frac{2}{\pi} \int_0^{1/2\pi} \cos(\lambda r \sin\theta)\, d\theta \tag{2.7}$$

relations (2.3) and (2.4) may be reduced to Schlömilch's integral equations

$$\int_0^{1/2\pi} \Big\{\varphi(r\sin\theta) - \frac{2}{\pi}\int_0^a \varphi(t)\,[H_1(t + r\sin\theta) + H_1(t - r\sin\theta) - H_1(a + r\sin\theta) - H_1(a - r\sin\theta)]\, dt - \frac{2}{\pi}\int_0^a \psi(t)\,[G_1(t + r\sin\theta) + G_1(t - r\sin\theta)]\, dt\Big\} d\theta = 2f_1(r) \tag{2.8}$$

$$\int_0^{1/2\pi} \Big\{\psi(r\sin\theta) - \frac{2}{\pi}\int_0^a \psi(t)\,[G_2(t + r\sin\theta) + G_2(t - r\sin\theta)]\, dt - \frac{2}{\pi}\int_0^a \varphi(t)\,[H_2(t + r\sin\theta) + H_2(t - r\sin\theta) - H_2(a + r\sin\theta) - H_2(a - r\sin\theta)]\, dt\Big\} d\theta = -2f_2(r) \tag{2.9}$$

where

$$H_{1,2}(x) = \int_0^\infty h_{1,2}(\lambda) \cos \lambda x d\lambda, \qquad G_{1,2}(x) = \int_0^\infty g_{1,2}(\lambda) \cos \lambda x d\lambda \tag{2.10}$$

After calculation we obtain

$$H_1 = 4h^3 \frac{12h^2 - x^2}{(4h^2 + x^2)^3}, \quad G_1 = Gh^3 \frac{16h^4 - 24x^2h^2 + x^4}{(4h^2 + x^2)^4}$$
$$H_2 = h\, \frac{4h^2 - x^2}{(4h^2 + x^2)^3} \qquad G_2 = 2h\, \frac{8h^4 - 2h^2x^2 + x^4}{(4h^2 + x^2)^3} \tag{2.11}$$

Solving Equations (2.8) and (2.9), we arrive at a system of Fredholm integral equations

$$\varphi(x) - \frac{2}{\pi}\int_0^a \varphi(t)\, K_1(x, t)\, dt - \frac{2}{\pi}\int_0^a \psi(t)\, L_1(x, t)\, dt = \frac{4}{\pi}\Big[f_1(0) + x\int_0^{1/2\pi} f_1'(x\sin\theta)\, d\theta\Big] \qquad (0 < x < a) \tag{2.12}$$

$$\psi(x) - \frac{2}{\pi}\int_0^a \psi(t)\, L_2(x, t)\, dt - \frac{2}{\pi}\int_0^a \varphi(t)\, K_2(x, t)\, dt = -\frac{4}{\pi}\Big[f_2(0) + x\int_0^{1/2\pi} f_2'(x\sin\theta)\, d\theta\Big] \qquad ((0 < x < a) \tag{2.13}$$

the kernels of which are given by Equations

$$
\begin{aligned}
K_1(x,t) &= H_1(t+x) + H_1(t-x) - H_1(a+x) - H_1(a-x)\\
L_1(x,t) &= G_1(t+x) + G_1(t-x)\\
K_2(x,t) &= H_2(t+x) + H_2(t-x) - H_2(a+x) - H_2(a-x)\\
L_2(x,t) &= G_2(t+x) + G_2(t-x)
\end{aligned} \qquad (2.14)
$$

Condition (1.19) may be reduced to the form

$$\int_4^a \psi(t)\,dt = 0 \qquad (2.15)$$

Thus, the solution to the problem set is given by Equations (1.11),(1.12), (1.14),(1.15),(2.1),(2.12),(2.13).

**3. Some numerical results.** We now turn to the special case, where the boundary of the half-space is free of stress, and to the edges of the crack are applied a uniform normal stress of intensity $q$. At the same time $\sigma_0 = \tau_0 = \tau_1 = \tau_2 = 0$, $\sigma_1 = \sigma_2 = -q$, from which

$$f_1(r) = -q, \qquad f_2(r) = c \qquad (3.1)$$

Introducing the dimensionless variables

$$x/a = \xi, \qquad t/a = \tau \qquad (3.2)$$

$$\varphi(x) = -\frac{4}{\pi}[q\chi_1(\xi) + c\chi_2(\xi)], \qquad \psi(x) = -\frac{4}{\pi}[q\omega_1(\xi) + c\omega_2(\xi)]$$

we reduce the problem to the solution of two systems of Fredholm integral equations

$$
\begin{aligned}
\chi_1(\xi) &= 1 + \int_0^1 M_1(\xi,\tau)\chi_1(\tau)\,d\tau + \int_0^1 N_1(\xi,\tau)\omega_1(\tau)\,d\tau\\
\omega_1(\xi) &= \int_0^1 N_2(\xi,\tau)\omega_1(\tau)\,d\tau + \int_0^1 M_2(\xi,\tau)\chi_1(\tau)\,d\tau
\end{aligned} \qquad (3.3)
$$

$$
\begin{aligned}
\chi_2(\xi) &= \int_0^1 M_1(\xi,\tau)\chi_2(\tau)\,d\tau + \int_0^1 N_1(\xi,\tau)\omega_2(\tau)\,d\tau\\
\omega_2(\xi) &= 1 + \int_0^1 N_2(\xi,\tau)\omega_2(\tau)\,d\tau + \int_0^1 M_2(\xi,\tau)\chi_2(\tau)\,d\tau
\end{aligned} \qquad (3.4)
$$

In relations (3.3) and (3.4) we introduce the notation

$$M_1 = \frac{8\beta^3}{\pi}[S_1(\tau+\xi) + S_1(\tau-\xi) - S_1(1+\xi) - S_1(1-\xi)],$$

$$N_1 = \frac{12\beta^3}{\pi}[R_1(\tau+\xi) + R_1(\tau-\xi)] \qquad (3.5)$$

$$M_2 = \frac{2\beta}{\pi}[S_2(\tau+\xi) + S_2(\tau-\xi) - S_2(1+\xi) - S_2(1-\xi)],$$

$$N_2 = \frac{4\beta}{\pi}[R_2(\tau+\xi) + R_2(\tau-\xi)]$$

$$S_1(u) = \frac{12\beta^2 - u^2}{(4\beta^2+u^2)^3}, \qquad R_1(u) = \frac{16\beta^4 - 24\beta^2u^2 + u^4}{(4\beta^2+u^2)^4} \qquad (3.6)$$

$$S_2(u) = \frac{4\beta^2 - u^2}{(4\beta^2+u^2)^2}, \qquad R_2(u) = \frac{8\beta^4 - 2\beta^2u^2 + u^4}{(4\beta^2+u^2)^3}$$

and put

$$\beta = h/a \qquad (3.7)$$

Equation (2.15) gives the following expression for the value $c$ :

$$c = -q\left(\int_0^1 \omega_1(\tau)\,d\tau\right)\left(\int_0^1 \omega_2(\tau)\,d\tau\right)^{-1} \tag{3.8}$$

By means of substitution of the system (3.3), (3.4) by the algebraic system, an approximate solution in the trapezoidal formula was obtained, where the interval of integration was partitioned into ten parts. Results of the calculation for two values of the parameter $\beta$ are shown in the following table.

| | $\beta = 1/2$ | | | | $\beta = 1$ | | | |
|---|---|---|---|---|---|---|---|---|
| $\xi$ | $\chi_1$ | $\omega_1$ | $\chi_2$ | $\omega_2$ | $\chi_1$ | $\omega_1$ | $\chi_2$ | $\omega_2$ |
| 0 | 4.49 | 1.54 | 0.682 | 1.81 | 1.50 | 0.187 | 0.462 | 1.40 |
| 0.1 | 4.37 | 1.49 | 0.663 | 1.80 | 1.49 | 0.184 | 0.457 | 1.40 |
| 0.2 | 4.07 | 1.38 | 0.612 | 1.77 | 1.46 | 0.176 | 0.440 | 1.40 |
| 0.3 | 3.61 | 1.20 | 0.541 | 1.73 | 1.42 | 0.164 | 0.412 | 1.39 |
| 0.4 | 2.95 | 0.968 | 0.440 | 1.70 | 1.37 | 0.148 | 0.375 | 1.37 |
| 0.5 | 2.21 | 0.695 | 0.338 | 1.65 | 1.30 | 0.129 | 0.329 | 1.37 |
| 0.6 | 1.46 | 0.415 | 0.212 | 1.60 | 1.23 | 0.108 | 0.277 | 1.34 |
| 0.7 | 0.729 | 0.133 | 0.125 | 1.54 | 1.16 | 0.0851 | 0.220 | 1.32 |
| 0.8 | 0.0420 | —0.111 | —0.024 | 1.47 | 1.08 | 0.0623 | 0.160 | 1.30 |
| 0.9 | —0.463 | —0.323 | —0.139 | 1.42 | 1.01 | 0.0403 | 0.0997 | 1.27 |
| 1 | —0.800 | —0.446 | —0.262 | 1.36 | 0.954 | 0.0199 | 0.0199 | 1.25 |

It is important to note that certain characteristics of the stress-deformation state may be expressed immediately by means of functions $\chi_{1,2}$ and $\omega_{1,2}$ . In particular, an asymptotic expression for formal stresses in the plane $z = 0$ as $r \to +a$ is

$$\sigma_0 = -\frac{1}{2\sqrt{r^2 - a^2}}\int_0^a \varphi(t)\,dt + O(1) \tag{3.9}$$

Referring this quantity to its value $\sigma_0^\infty$ in the limiting case $h \to \infty$ , corresponding to a crack in an unlimited body [4], we find

$$\frac{\sigma_0}{\sigma_0^\infty} = \gamma + O\left(\frac{r^2}{a^2} - 1\right)^{1/2} \tag{3.10}$$

$$\gamma = \int_0^1 \chi_1(\xi)\,d\xi - \left(\int_0^1 \omega_1(\xi)\,d\xi\right)\left(\int_0^1 \omega_2(\xi)\,d\xi\right)^{-1}\left(\int_0^1 \chi_2(\xi)\,d\xi\right) \tag{3.11}$$

The values of the coefficient $\gamma$ , characterizing the degree of increase of stress concentration connected with the presence of free boundary of the half-space, turn out to be the following:

$$\gamma = 2.19 \quad \text{for} \quad \beta = 1/2, \qquad \gamma = 1.27 \quad \text{for} \quad \beta = 1$$

For obtaining the approximate solution to the problem under consideration for large values of $h/a$ , the method of expansion in a series of the small parameter $\alpha = a/h$ may be applied.

After performing the corresponding calculations the following expressions for the basic unknown functions may be obtained:

$$\varphi(x) = -\frac{4q}{\pi}\left[1 + \frac{5}{3\pi}\alpha^3 - \frac{21}{20\pi}(1 + 5\xi^2)\alpha^5 + O(\alpha^7)\right]$$

$$\psi(x) = -\frac{16q}{3\pi}\left[\frac{5}{16\pi}(1 - 3\xi^2)\alpha^5 + O(\alpha^7)\right] \tag{3.12}$$

The coefficient $\gamma$ may be calculated by Equation

$$\gamma = 1 + \frac{5}{3\pi}\alpha^3 - \frac{14}{5\pi}\alpha^5 + O(\alpha^7) \tag{3.13}$$

which for $\alpha = \frac{1}{2}$ and $\alpha = \frac{1}{3}$ gives $\gamma = 1.04$ and $\gamma = 1.02$, respectively; so that for $h/a \approx 2-3$ the presence of a free boundary has practically no effect on the stress concentration factor.

The method of expansion in a small parameter may also be applied in the problem of deformation of an unbounded body weakened by two plane circular cracks (cf. paper [2]). In this case for the coefficient $\gamma$ we obtain the following formula:

$$\gamma = 1 - \frac{2}{3\pi}\alpha^3 + \frac{4}{5\pi}\alpha^5 + O(\alpha^7) \tag{3.14}$$

it obviously shows that there occurs a reduction of the stress concentration in a comparison with the case of a single crack.

In conclusion we will point out that the method developed in the present paper may also be applied to the case where the boundary of the half-space is rigidly clamped. For this, in place of the first two conditions (1.3) we should assume

$$u_z|_{z=-h} = u_r|_{z=-h} = 0 \tag{3.15}$$

The presence of the latter condition necessitates finding the function $\sigma$ in place of the function $\Phi$ (cf. (1.1)). This somewhat complicates the calculations, however the problem, as before, may be reduced to the system of regular Fredholm equations. One must note that in contrast to the case of a free boundary or two cracks, for a fixed boundary both the kernel of the Fredholm equations and the other characteristics (for example the coefficient $\gamma$ cf. (3.10)) turn out to be dependent on Poisson's ratio $\nu$. We only mention here the equation for $\gamma$ in the form of an expansion in powers of $\alpha$

$$\gamma = 1 - \frac{\varkappa^2 + 19}{12\pi\varkappa}\alpha^3 + \frac{\varkappa^2 + 41}{15\pi\varkappa}\alpha^5 + O(\alpha^7) \qquad (\varkappa = 3 - 4\nu) \tag{3.16}$$

For $\nu = \frac{1}{3}$, $\alpha = \frac{1}{2}$ we find $\gamma = 0.97$.

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